Integral solution for diffraction problems involving conducting surfaces with complex geometries. I. Theory

Mohamed F. El-Hewie and Richard J. Cook

Frank J. Seiler Research Laboratory, U.S. Air Force Academy, Colorado Springs, Colorado 80840-6528

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For an arbitrary conducting surface $Z(x, y)$ we obtained an analytical expression for the local refractive index ν as a function of Z, $\partial Z/\partial x$, $\partial Z/\partial y$, and the Drude conductivity σ by using the complex ray-tracing method. The Fresnel coefficients of reflectance and transmittance are then employed, and the value of ν is obtained to determine the scattered and refracted fields. The proposed method has advantages over the methods of solution by the Debye potential, the Laplace transform, and the vector-wave equation in the computation of the scattering and absorption parameters of a wide range of complex surface and wave-front geometries. The surface integrals obtained in the present study include the surface function in an explicit and concise form.

1. INTRODUCTION

Since Mie (1908) introduced his rigorous solution of Maxwell's equations for the diffraction of a plane monochromatic wave by a conducting sphere by using Debye's potentials, a great number of applications have been made by using his approach in solving diffraction problems involving particles of various shapes.[1] The main drawback of using the Debye potential method is the difficulty of finding suitable solutions for the scalar-wave equation to represent either an incident wave front other than a plane wave or the scattered and refracted waves in complex geometries. Another approach to the solution of the diffraction problems is to solve the vector-wave equation in the space external to the boundary surface. The solution obtained in this way is a function of the integral of the spherical vector harmonics at the surface.[2] A third approach, in the long-wavelength range, is the solution of the Stratton–Chu–Silver integral (SCSI) form with the perfectly conducting surface and tangent plane approximations.[3,4]

In the visible range of wavelength the conductivity is finite. Leader[5] worked out the scattering by a rough surface and obtained a solution of the SCSI form with the derivations of the Fresnel coefficients not expressed explicitly in terms of the surface function $Z(x, y)$. Also, Bahar and Rajan[6] analyzed the same problem by using a complicated mathematical formulation.

In the present paper we present a solution palatable to workers in the field of optics. We have utilized the generalized ray-tracing procedure developed by Kneisly[7] and others[8,9] to analyze the diffraction of the electromagnetic waves at an arbitrary surface of a finite conductivity. The definition of the refractive index for a conducting medium used by Stratton[10] for a plane surface is generalized for an arbitrary surface. The Fresnel coefficients of reflectance and transmittance are then employed with the new geometry- and medium-dependent refractive index in the place of the traditional only-medium-dependent refractive index.

We will ignore internal reflection in the conducting medium, since scattering is greatly overwhelmed by absorption. This simplifies the boundary conditions to a great extent. Our analysis is also restricted to geometries in which no multiple reflection occurs between opposing surfaces. As we will see in the application of the present method, the Mie problem is a good candidate for our method of solution because, for a metallic sphere, it is realistic to neglect both internal and external multiple scattering. Treatment of penetrable and inhomogeneous bodies was carried out by others for simpler surface geometries by using different methods of numerical computation.[11–13] A study was done for refracted fields from an arbitrary refractive surface,[14] but reflection was ignored in that study.

In the applications in which the value of the field at the surface is the main concern, such as in laser–solid interaction, our solution is expressed in terms of modified Fresnel coefficients. In the applications in which the scattered field is desired, such as in radar measurements and remote imaging applications, the solution is expressed in a modified SCSI form.

The rest of the paper falls into six sections. In Section 2, we obtain an analytical expression of the refractive index ν in terms of the surface $Z(x, y)$, Z_x, Z_y, and the physical properties of the media. In Section 3 we determine the different electromagnetic field vectors of the locally polarized waves on the surface $Z(x, y)$. In Section 4 the absorption of the incident energy is analyzed. In Section 5 the scattering fields are described. In Section 6 an application to a spherical wave front converging on a finitely conducting surface is discussed. Our conclusion is stated in Section 7.

2. REFRACTIVE INDEX OF AN ARBITRARY CONDUCTING SURFACE

The Stratton definition of the refractive index of a conducting plane surface can be generalized to a surface of an arbitrary coordinate function as follows. At a surface of discontinuity $Z(x, y)$ separating two media, the local normal to the surface, $\hat{n}$, makes a polar angle θ with the z axis and an azimuth φ with the x axis in the xy plane of the reference frame. These angles are given by differential geometry principles as (see also Ref. 3 and 4)

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$$\tan\theta = -(Z_x{}^2 + Z_y{}^2)^{1/2}, \tag{1a}$$

$$\tan\varphi = Z_y/Z_x, \tag{1b}$$

where

$$Z_x = \partial Z/\partial x, \qquad Z_y = \partial Z/\partial y. \tag{1c}$$

The subscripts x and y denote differentiation only when they appear on Z; they denote Cartesian components elsewhere. The unit normal vector $\hat{n}$,

$$\hat{n} = (-Z_x\hat{i} - Z_y\hat{j} + k)/C, \tag{2a}$$

is then determined, where

$$C = (1 + Z_x{}^2 + Z_y{}^2)^{1/2}. \tag{2b}$$

The propagation unit vector normal to the incoming wave front $\hat{n}_i$ is given by

$$\hat{n}_i = \sin\theta_i\cos\varphi_i\hat{i} + \sin\theta_i\sin\varphi_i\hat{j} + \cos\theta_i\hat{k}, \tag{2c}$$

where θ_i and φ_i are given independent of the position vector of a point at the surface, $\mathbf{r}$, for a plane-wave illumination. At a point $P(x, y, Z)$ on the surface Z, the unit vectors along the refracted and reflected rays are of well-known form [see, for example, Eqs. (22) and (29) of Ref. 7] and are, respectively, given by

$$\hat{n}_r = (k_i/k_r)\hat{n}_i + [\cos(\theta_{rn} + \pi) - (k_i/k_r)\cos\theta_{\text{in}}]\hat{n} \tag{2d}$$

and

$$\hat{n}_s = \hat{n}_i - 2\cos\theta_{\text{in}}\hat{n}. \tag{2e}$$

π is added to θ_{rn} because the refracted rays are propagating away from the positive z direction. The subscripts i, s, and r refer to quantities related to the incident, reflected, and refracted waves, respectively. Double subscripts on angles indicate the angle between the propagating vectors of the designated waves and the surface normal; e.g., θ_{rn} is the angle between $\hat{n}_r$ and $\hat{n}$. k_i and k_r are, respectively, the wave propagation numbers of the incident and the refracted waves. k_i is real, whereas k_r is complex. The local angles of incidence, $\theta_{\text{in}}(x, y, Z)$ and $\varphi_{\text{in}}(x, y, Z)$, are given by

$$\varphi_{\text{in}}(x, y, Z) = \varphi_{rn} = \varphi_{sn} = \varphi(x, y, Z) - \varphi_i(x, y, Z), \tag{3a}$$

$$\begin{aligned}\cos\theta_{\text{in}} &= \hat{n}\cdot\hat{n}_i \\ &= (-Z_x\sin\theta_i\cos\varphi_i - Z_y\sin\theta_i\sin\varphi_i + \cos\theta_i)/C\end{aligned} \tag{3b}$$

and

$$\begin{aligned}\sin\theta_{\text{in}} &= [1 - (\hat{n}\cdot\hat{n}_i)^2]^{1/2} \\ &= \{1 - [(-Z_x\sin\theta_i\cos\varphi_i - Z_y\sin\theta_i\sin\varphi_i \\ &\quad + \cos\theta_i)/C]^2\}^{1/2}.\end{aligned} \tag{3c}$$

For the refracted waves, Snell's law in its scalar form gives the local angle of refraction, $\theta_{rn}(x, y, Z)$, as

$$k_r\sin\theta_{rn} = k_i\sin\theta_{\text{in}}. \tag{4}$$

The arguments x, y, and Z are sometimes suppressed to simplify notation whenever there is no chance for confusion.

An aplanar wave front is described by a ray-aberration vector $\mathbf{u}(u_x, u_y, u_z)$. The components of $\mathbf{u}$ are the projections of the phase shift along an incident ray on the different Cartesian directions. Now, consider the following general forms for the incident, refracted, and scattered waves[1]:

$$\mathbf{e}_i(r, t) = \mathbf{e}_i(r)\exp[i(k_i\hat{n}_i\cdot\mathbf{r} + k_i\mathbf{u}\cdot\mathbf{r} - \omega t)], \tag{5a}$$

$$\mathbf{h}_i(r, t) = \mathbf{h}_i(r)\exp[i(k_i\hat{n}_i\cdot\mathbf{r} + k_i\mathbf{u}\cdot\mathbf{r} - \omega t)], \tag{5b}$$

$$\mathbf{e}_r(r, t) = \mathbf{e}_r(r)\exp[i(k_r\hat{n}_r\cdot\mathbf{r} + k_i\mathbf{u}\cdot\mathbf{r} - \omega t)], \tag{5c}$$

$$\mathbf{h}_r(r, t) = \mathbf{h}_r(r)\exp[i(k_r\hat{n}_r\cdot\mathbf{r} + k_i\mathbf{u}\cdot\mathbf{r} - \omega t)], \tag{5d}$$

$$\mathbf{e}_s(r, t) = \mathbf{e}_s(r)\exp[i(k_s\hat{n}_s\cdot\mathbf{r} + k_i\mathbf{u}\cdot\mathbf{r} - \omega t)], \tag{5e}$$

$$\mathbf{h}_s(r, t) = \mathbf{h}_s(r)\exp[i(k_s\hat{n}_s\cdot\mathbf{r} + k_i\mathbf{u}\cdot\mathbf{r} - \omega t)], \tag{5f}$$

with $\mathbf{e}(r)$ and $\mathbf{h}(r)$ terms real. $\mathbf{e}_r(r, t)$, $\mathbf{h}_r(r, t)$, $\mathbf{e}_s(r, t)$, and $\mathbf{h}_s(r, t)$ are the refracted and reflected fields and are related by the Maxwell curl equations,

$$\mu_r\dot{\mathbf{h}}_r(r, t) = -\nabla\times\mathbf{e}_r(r, t) \tag{6a}$$

and

$$\mu_i[\dot{\mathbf{h}}_i(r, t) + \dot{\mathbf{h}}_s(r, t)] = -\nabla\times[\mathbf{e}_i(r, t) + \mathbf{e}_s(r, t)]. \tag{6b}$$

On using Eqs. (5) in Eqs. (6), with some manipulation, we get the following two expressions for the refracted and reflected magnetic fields:

$$\mathbf{h}_r = \nabla(k_r\hat{n}_r\cdot\mathbf{r}/k_i + \mathbf{u}\cdot\mathbf{r})\times\mathbf{e}_r/\eta, \tag{7a}$$

$$\mathbf{h}_s = \mathbf{h}_i + [\nabla(\hat{n}_i\cdot\mathbf{r} + u\cdot\mathbf{r})\times\mathbf{e}_i + \nabla(\hat{n}_s\cdot\mathbf{r} + \mathbf{u}\cdot\mathbf{r})\times\mathbf{e}_s]/\eta \tag{7b}$$

since the curl of the amplitudes of the incident, reflected, and scattered fields vanish at the boundary surface. The angular frequency of light $\omega = k_i/(\epsilon_i\mu_i)^{1/2}$ and the intrinsic impedance of free space $\eta = (\mu_i/\epsilon_i)^{1/2}$ are used. μ_i and ϵ_i are, respectively, the magnetic permeability and the electric permittivity of the free space.

The real and imaginary parts of the refracted wave propagation vector $k_r\hat{n}_r$ are obtained as follows.[10] k_r and $\cos\theta_{rn}$ are written explicitly in their complex forms as

$$k_r = \xi + i\zeta \tag{8a}$$

and

$$\cos\theta_{rn} = \Psi + i\Lambda, \tag{8b}$$

where ξ and ζ are known from the elementary electron theory of the optics of metals and are conveniently given by[1]

$$\xi = \omega\{0.5\mu_r\epsilon_r[(1 + \sigma_r{}^2/\epsilon_r{}^2\omega^2)^{1/2} + 1]\}^{1/2} \tag{8c}$$

and

$$\zeta = \omega\{0.5\mu_r\epsilon_r[(1 + \sigma_r{}^2/\epsilon_r{}^2\omega^2)^{1/2} - 1]\}^{1/2}. \tag{8d}$$

σ_r is the Drude electrical conductivity of the metal. Ψ and Λ are functions of Z, Z_x, Z_y, θ_i, φ_i, and the physical properties of the media and are obtained from Eq. (4). After squaring Eq. (4), using Eq. (8a), and performing some manipulation, we get

$$\Psi = \{0.5[(A^2 + B^2)^{1/2} + A]\}^{1/2} \tag{8e}$$

and

$$\Lambda = \{0.5[(A^2 + B^2)^{1/2} - A]\}^{1/2}, \quad (8f)$$

where

$$A = 1 - k_i^2 \sin^2\theta_{in}(\xi^2 - \zeta^2)/(\xi^2 + \zeta^2)^2 \quad (8g)$$

and

$$B = 2k_i^2(\xi\zeta)\sin^2\theta_{in}/(\xi^2 + \zeta^2)^2. \quad (8h)$$

After substituting $\sin\theta_{in}$ from Eq. (3c) into Eqs. (8g) and (8h), we get

$$A = 1 - k_i^2[(\xi^2 - \zeta^2)/(\xi^2 + \zeta^2)^2]\{1 - [(-Z_x \sin\theta_i \cos\varphi_i - Z_y \sin\theta_i \sin\varphi_i + \cos\theta_i)/C]^2\} \quad (8i)$$

and

$$B = 2k_i^2[\xi\zeta/(\xi^2 + \zeta^2)^2]\{1 - [(-Z_x \sin\theta_i \cos\varphi_i - Z_y \sin\theta_i \sin\varphi_i + \cos\theta_i)/C^2\}. \quad (8j)$$

After substituting $\cos\theta_{rn}$ from Eq. (8b) into Eq. (2c), we get

$$\hat{n}_r = (k_i/k_r)\hat{n}_i - [\Psi + i\Lambda + (k_i/k_r)\cos\theta_{in}]\hat{n}. \quad (9a)$$

By multiplying both sides of Eq. (9a) by k_r from Eq. (8a) and then arranging, we get

$$k_r\hat{n}_r = k_i\hat{n}_i - (\xi\Psi - \zeta\Lambda + k_i\cos\theta_{in})\hat{n} - i(\zeta\Psi + \xi\Lambda)\hat{n} = k_i[n_i + (q + ip)\hat{n}]. \quad (9b)$$

After substituting $\cos\theta_{in}$ from Eq. (3b) into Eq. (9b), we get

$$q = -(\xi\Psi - \zeta\Lambda)/k_i + (Z_x \sin\theta_i \cos\varphi_i + Z_y \sin\theta_i \sin\varphi_i - \cos\theta_i)/C \quad (10a)$$

and

$$p = -(\zeta\Psi + \xi\Lambda)/k_i. \quad (10b)$$

The q and p defined by Eqs. (10) for an arbitrary surface are similar to those obtained by Stratton for a plane surface, but they are now functions of Z and its derivatives. After substituting $k_r\hat{n}_r$ from Eq. (9b) into Eq. (7a), we get

$$\mathbf{h}_r = \nabla[(\hat{n}_i + (q + ip)\hat{n} + \mathbf{u}) \cdot \mathbf{r}] \times \mathbf{e}_r/\eta. \quad (11)$$

Equation (11) relates the refracted magnetic and electric fields algebraically, since the gradient term is only a function of the surface geometry and parameters. For an incident plane wave, $\hat{n}_r$ depends on $\mathbf{r}$ through $\hat{n}$ [Eq. (2d)]. For an arbitrary wave front, $\hat{n}_r$ depends on $\mathbf{r}$ through both $\hat{n}_i$ and $\hat{n}$. Equation (11) is a direct result of our assumption that only a single refracted wave exists inside the conducting medium, which simplifies the boundary conditions and makes it possible to determine the surface values of the field algebraically. Our assumption is justified on the basis that the skin depth in metals is ultimately small compared with the wavelength of the field. This is not the case with dielectrics, where $\mathbf{e}_r$ is a sum of an ingoing wave and an internally reflected wave.

Equation (11) represents a system of inhomogeneous waves.[10] The imaginary part of the gradient term in Eq. (11) represents the attenuation of the conducting medium. The real part of the gradient term is the local wave propagation vector along the refracted ray and can be written as

$$\mathbf{K} = \hat{n}_i + q\hat{n} + \mathbf{u}. \quad (12)$$

The real local angle of refraction θ_{rl} is then defined as

$$\cos\theta_{rl} = \mathbf{K} \cdot \hat{n}/|\mathbf{K}| \quad (13a)$$

and

$$\sin\theta_{rl} = [1 - (\mathbf{K} \cdot \hat{n}/|\mathbf{K}|)^2]^{1/2}. \quad (13b)$$

By substituting $\hat{n}_i$ and $\hat{n}$ from Eqs. (2) and $\mathbf{K}$ from Eq. (12) into Eqs. (13), we get

$$\cos\theta_{rl} = (-Z_xF_1 - Z_yF_2 + F_3 + qC)/[(CF_1 - qZ_x)^2 + (CF_2 - qZ_y)^2 + (CF_3 + q)^2]^{1/2} \quad (13c)$$

and

$$\sin\theta_{rl} = \{1 - (-Z_xF_1 - Z_yF_2 + F_3 + qC)^2/[(CF_1 - qZ_x)^2 + (CF_2 - qZ_y)^2 + (CF_3 + q)^2]\}^{1/2}, \quad (13d)$$

where

$$F_1 = \sin\theta_i \cos\varphi_i + u_x, \quad (14a)$$

$$F_2 = \sin\theta_i \sin\varphi_i + u_y, \quad (14b)$$

and

$$F_3 = \cos\theta_i + u_z. \quad (14c)$$

The u's vanish for a plane wave and should be given for an aberrated wave. From Eqs. (3c), (13d,) and (14), the local refractive index is given as

$$\nu(Z, \sigma, \omega) = \sin\theta_{in}/\sin\theta_{rl} = [(1 - \{[-Z_x(F_1 - u_x) - Z_y(F_2 - u_y) + F_3 - u_z]/C\}^2)/\{1 - (-Z_xF_1 - Z_yF_2 + F_3 + qC)^2/[(CF_1 - qZ_x)^2 + (CF_2 - qZ_y)^2 + (CF_3 + q)^2]\}]^{1/2}. \quad (15)$$

Equation (15) gives the local refractive index in terms of Z and its derivatives, the angles of incidence with respect to a reference frame, the optical abberation $\mathbf{u}$, and the physical properties of the media. Equation (15) can be reduced to the case of a plane wave irradiating a plane-conducting surface as follows. Let the surface derivatives Z_x and Z_y vanish, and let $\varphi_i = 0$ and $\mathbf{u} = 0$; we get from the above equations

$$F_1 = \sin\theta_i, \quad F_2 = 0, \quad F_3 = \cos\theta_i, \quad C = 1,$$
$$q = -(\xi\Psi - \zeta\Lambda)/k_i - \cos\theta_i. \quad (16a)$$

Substituting the different values from Eq. (16a) into Eq. (15) gives

$$\nu_{\mathrm{Stratton}} = [k_i^2 \sin^2\theta_i + (\xi\Psi - \zeta\Lambda)^2]^{1/2}/k_i, \quad (16b)$$

which was obtained by Stratton[10] with the term $(\xi\Psi - \zeta\Lambda)$ equivalent to his q.

3. ELECTROMAGNETIC FIELD VECTORS OF THE LOCALLY POLARIZED WAVES ON THE SURFACE $Z(x, y)$

Let $\hat{a}$ be the polarization unit vector and let E_i be the incident electric field strength. Also let $\hat{a}$ make an angle α with the plane that contains both the unit vectors $\hat{n}_i$ (along the incident ray) and $\hat{k}$ (along the z axis). From an orthogonal system of unit vectors, $\hat{a}$ can be written in the following Cartesian form (see Fig. 1 and Refs. 5 and 15)

$$\begin{aligned}\hat{a} = {} & (-\cos\alpha\cos\theta_i\cos\varphi_i - \sin\alpha\sin\varphi_i)\hat{i} \\ & + (-\cos\alpha\cos\theta_i\sin\varphi_i + \sin\alpha\cos\varphi_i)\hat{j} \\ & + \cos\alpha\sin\theta_i\hat{k}.\end{aligned} \tag{17a}$$

At any local point at the surface Z the incident fields can be decomposed into both a locally polarized s wave (TE) and a p wave (TM) as follows. A local orthogonal system of unit vectors is defined as

$$\hat{t} = (\hat{n}_i \times \hat{n})/[1 - (\hat{n}_i \cdot \hat{n})^2]^{1/2}, \tag{17b}$$

$$\hat{p} = \hat{n} \times \hat{t}, \tag{17c}$$

and

$$\hat{d} = \hat{n}_i \times \hat{t}. \tag{17d}$$

After substituting $\hat{n}_i$ and $\hat{n}$ from Eq. (2) into Eqs. (17b)–(17d) and executing the vectorial multiplication, we finally get

$$\begin{aligned}\hat{t} = {} & (1/C\sin\theta_{in})[\hat{i}(\sin\theta_i\sin\varphi_i + Z_y\cos\theta_i) \\ & + \hat{j}(-\sin\theta_i\cos\varphi_i - Z_x\cos\theta_i) \\ & + \hat{k}\sin\theta_i(Z_x\sin\varphi_i - Z_y\cos\varphi_i)],\end{aligned} \tag{18a}$$

$$\begin{aligned}\hat{p} = {} & (1/C^2\sin\theta_{in})\{\hat{i}[\sin\theta_i(Z_y{}^2\cos\varphi_i - Z_xZ_y\sin\varphi_i \\ & + \cos\varphi_i) + Z_x\cos\theta_i] + \hat{j}[\sin\theta_i(Z_x{}^2\sin\varphi_i \\ & - Z_xZ_y\cos\varphi_i + \sin\varphi_i) + Z_y\cos\theta_i] + \hat{k}[\cos\theta_i(Z_x{}^2 \\ & + Z_y{}^2) + \sin\theta_i(Z_x\cos\varphi_i + Z_y\sin\varphi_i)]\},\end{aligned} \tag{18b}$$

$$\begin{aligned}\hat{a}\cdot\hat{t} = {} & (-1/C\sin\theta_{in})[Z_x(\sin\alpha\cos\theta_i\cos\varphi_i \\ & - \cos\alpha\sin\varphi_i) + Z_y(\sin\alpha\cos\theta_i\sin\varphi_i \\ & + \cos\alpha\cos\varphi_i) + \sin\alpha\sin\theta_i],\end{aligned} \tag{18c}$$

$$\begin{aligned}\hat{a}\cdot\hat{n} = {} & (1/C)[Z_x(\cos\alpha\cos\theta_i\cos\varphi_i + \sin\alpha\sin\varphi_i) \\ & + Z_y(\cos\alpha\cos\theta_i\sin\varphi_i - \sin\alpha\cos\varphi_i) \\ & + \cos\alpha\sin\theta_i],\end{aligned} \tag{18d}$$

and

$$\begin{aligned}\hat{a}\cdot\hat{p} = {} & (1/C^2\sin\theta_{in})\{(-\cos\alpha\cos\theta_i\cos\varphi_i \\ & - \sin\alpha\sin\varphi_i)[\sin\theta_i(Z_y{}^2\cos\varphi_i - Z_xZ_y\sin\varphi_i \\ & + \cos\varphi_i) + Z_x\cos\theta_i] + (-\cos\alpha\cos\theta_i\sin\varphi_i \\ & + \sin\alpha\cos\varphi_i)[\sin\theta_i(Z_x{}^2\sin\varphi_i - Z_xZ_y\cos\varphi_i \\ & + \sin\varphi_i) + Z_y\cos\theta_i] + \cos\alpha\sin\theta_i[\cos\theta_i(Z_x{}^2 + Z_y{}^2) \\ & + \sin\theta_i(Z_x\cos\varphi_i + Z_y\sin\varphi_i)]\}.\end{aligned} \tag{18e}$$

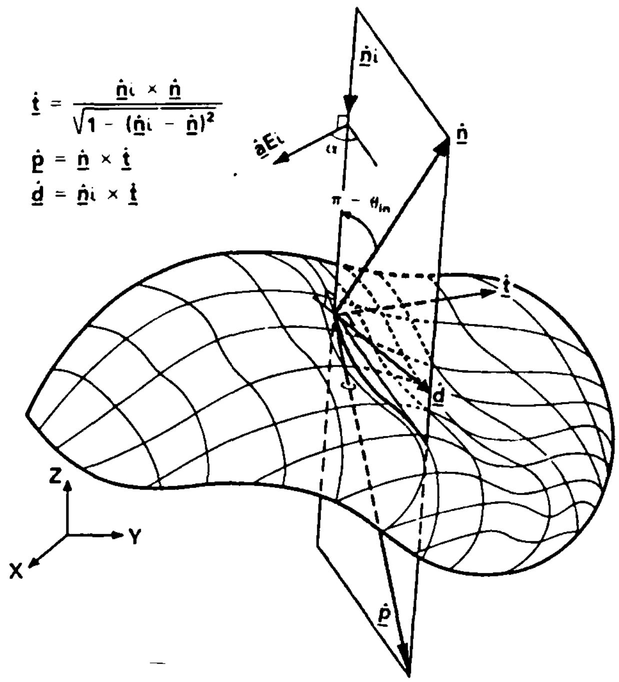

Fig. 1. Geometry of the irradiated surface.

For an s-polarized (TE) wave, the field vector components at the surface are then given by

$$\mathbf{e}_s^i = (\hat{a}\cdot\hat{t})\hat{t}E_i \tag{19a}$$

and

$$\mathbf{h}_s^i = \hat{n}_i \times \mathbf{e}_s^i/\eta, \tag{19b}$$

and for a p-polarized (TM) wave the field vector components at the surface are given by

$$\mathbf{e}_p^i = [(\hat{a}\cdot\hat{n})\hat{n} + (\hat{a}\cdot\hat{p})\hat{p}]E_i \tag{19c}$$

and

$$\mathbf{h}_p^i = \hat{n}_i \times \mathbf{e}_p^i/\eta. \tag{19d}$$

On $\mathbf{e}$ and $\mathbf{h}$ the subscript s denotes scattering, but on $\mathbf{e}^i$ and $\mathbf{h}^i$ it denotes an s wave (TE). The superscripts i and a (below) stand, respectively, for incidence and absorption. The amount of local incident energy flow per unit area normal to the surface in each wave is then defined as

$$\mathcal{J}_s^i = 0.5\mathbf{e}_s^i\cdot\mathbf{e}_s^{*i}\cos\theta_{in}/\eta \tag{20a}$$

and

$$\mathcal{J}_p^i = 0.5\mathbf{e}_p^i\cdot\mathbf{e}_p^{*i}\cos\theta_{in}/\eta. \tag{20b}$$

By substituting $\cos\theta_{in}$ from Eq. (3b) and the $\mathbf{e}^i$ terms from Eqs. (19) into Eqs. (20), we obtain expressions for the incident power intensity in terms of Z and its derivatives and the angles of incidence:

$$\begin{aligned}\mathcal{J}_s^i & = 0.5(E_i{}^2\cos\theta_{in}/\eta)(\hat{a}\cdot\hat{t})^2 \\ & = 0.5[E_i{}^2\cos\theta_{in}/\eta(C\sin\theta_{in})^2] \\ & \quad \times \{Z_x(\sin\alpha\cos\theta_i\cos\varphi_i - \cos\alpha\sin\varphi_i) \\ & \quad + Z_y(\sin\alpha\cos\theta_i\sin\varphi_i + \cos\alpha\cos\varphi_i) + \sin\alpha\sin\theta_i\}^2,\end{aligned} \tag{21}$$

$$J_p^i = 0.5(E_i^2 \cos\theta_{in}/\eta)[(\hat{a}\cdot\hat{n})^2 + (\hat{a}\cdot\hat{p})^2]. \quad (22)$$

All terms in Eq. (22) were obtained above in terms of the angles of incidence and the surface derivatives.

4. ABSORPTION

Equation (15) and Eqs. (21) and (22) give, respectively, the local refractive index and the local incident power intensity. Thus the amount of power absorbed per unit surface area is obtained from the absorptivities α_s and α_p, respectively, for the s and p waves as follows[1]:

$$\alpha_s = \sin 2\theta_{rl} \sin 2\theta_{in}/\sin^2(\theta_{in} + \theta_{rl}), \quad (23a)$$

$$\alpha_p = \sin 2\theta_{rl} \sin 2\theta_{in}/\sin^2(\theta_{in} + \theta_{rl})\cos^2(\theta_{in} - \theta_{rl}). \quad (23b)$$

After decomposing the different trigonometric functions in Eqs. (23) and substituting $(1 - \sin^2\theta_{in}/\nu^2)^{1/2}$ and $\sin\theta_{in}/\nu$, respectively, for $\cos\theta_{rl}$ and $\sin\theta_{rl}$, we finally get

$$\alpha_s(Z, \sigma, \omega) = 4\cos\theta_{in}(\nu^2 - \sin^2\theta_{in})^{1/2}/ [(\nu^2 - \sin^2\theta_{in})^{1/2} + \cos\theta_{in}]^2 \quad (24)$$

and

$$\alpha_p(Z, \sigma, \omega) = 4\nu^2\cos\theta_{in}(\nu^2 - \sin^2\theta_{in})^{1/2}/[(\nu^2 - \sin^2\theta_{in})^{1/2} + \cos\theta_{in}]^2[\cos\theta_{in}(\nu^2 - \sin^2\theta_{in})^{1/2} + \sin^2\theta_{in}]^{\cancel{1/2}\,2}. \quad (25)$$

Equations (24) and (25) give the local absorptivities for the s and p waves in terms of Z and its derivatives, the angles of incidence with respect to a reference frame, the optical aberration **u**, and the physical properties of the media. Thus the local net power absorbed is

$$J_{s+p}^a = \alpha_s J_s^i + \alpha_p J_p^i. \quad (26)$$

Integrating Eq. (26) over the surface area exposed to the incident rays gives the total absorbed power.

5. SCATTERING

Because the scattered wave front propagates in a nondissipative medium, interference dominates absorption. Therefore the electric and magnetic field vectors, and not the energy fluences, are the concerned variables. Equations (15) and (19) give, respectively, the local refractive index and the local incident field vectors. The Fresnel coefficients of reflectance R_s and R_p, respectively, for the s and p waves are then given by[1]

$$R_s = -\sin(\theta_{in} - \theta_{rl})/\sin(\theta_{in} + \theta_{rl}) \quad (27a)$$

and

$$R_p = \tan(\theta_{in} - \theta_{rl})/\tan(\theta_{in} + \theta_{rl}). \quad (27b)$$

In a way similar to that used with Eqs. (24) and (25) for the absorptivities, we get the following expressions for the Fresnel's reflection coefficients:

$$R_s = -[(\nu^2 - \sin^2\theta_{in})^{1/2} - \cos\theta_{in}]/[(\nu^2 - \sin^2\theta_{in})^{1/2} + \cos\theta_{in}], \quad (28)$$

$$R_p = -R_s/[\cos\theta_{in}(\nu^2 - \sin\theta_{in})^{1/2} - \sin^2\theta_{in}]. \quad (29)$$

Equations (28) and (29) give the reflection coefficients for the s and p waves in terms of Z and its derivatives, the angles of incidence with respect to a reference frame, the optical aberration **u**, and the physical properties of the media.

The far-zone scattered electric field $\mathbf{E}_s$ at a point in the space surrounding the irradiated surface is obtained by the SCSI as[4]

$$\mathbf{E}_s(P) = -[ik_i \exp(-ik_iR)/4\pi R]\hat{n}_p \times \int [\hat{n}\times\mathbf{e}_s - \eta\hat{n}_p(\hat{n}\times\mathbf{h}_s)] \times \exp(ik_i\mathbf{r}\cdot\hat{n}_p)dS, \quad (30)$$

where R is the distance from the origin to the point P, $\hat{n}_p$ is the unit vector in the direction of observation and dS is a surface element. The surface currents in Eq. (30) are defined by[4]

$$\hat{n}\times\mathbf{e}_s = \hat{n}\times\mathbf{e}_s^i(1 + R_s) + \eta(\hat{n}\cdot\hat{n}_i)(1 - R_p)\mathbf{h}_p^i \quad (31)$$

and

$$\hat{n}\times\mathbf{h}_s = \hat{n}\times\mathbf{h}_p^i(1 + R_p) - (\hat{n}\cdot\hat{n}_i)(1 - R_s)\mathbf{e}_s^i/\eta. \quad (32)$$

All terms in Eqs. (31) and (32) are fully determined by Eqs. (2), (19), (28), and (29). Thus the scattered field, Eq. (30), can be represented explicitly in terms of Z and its derivatives, the angles of incidence with respect to a reference frame, the optical aberration **u**, and the physical properties of the media.

6. APPLICATION TO A SPHERICAL WAVE FRONT CONVERGING ON A FINITELY CONDUCTING SURFACE

A spherical wave front converging on a conducting surface has a common concern in many optical fields, such as in inertial confinement fusion and atomic cooling by focused laser beams.

A spherical wave front can be produced by an aplanatic lens as follows. Let an aberration-free convex lens of spherical surfaces of revolution be located with its axis of revolution coinciding with the z axis. A plane wave propagating in the negative z direction will emerge from the lens as a spherical wave. The electromagnetic field vector at the focal volume of the lens is then given by[15]

$$\mathbf{E}_i = (E_0fk_i)\int_0^\gamma\int_0^{2\pi} \cos^{1/2}\theta \sin\theta\hat{a} \exp(ik_i\hat{n}_i\cdot\mathbf{r})d\theta d\varphi, \quad (33)$$

where γ is the semi-aperture angle of the lens, f is its focal length, and E_0 is the electric field strength at the lens. The solution of Eq. (33) is given in terms of the diffraction integrals I_1, I_2, and I_3 as[15]

$$E_{xi} = -iE_0fk_i(I_0 + I_2\cos 2\varphi), \quad (34a)$$

$$E_{yi} = -iE_0fk_iI_2\sin 2\varphi, \quad (34b)$$

$$E_{zi} = -2E_0fk_iI_1\cos\varphi, \quad (34c)$$

$$H_{xi} = -iE_0fk_iI_2\sin 2\varphi/\eta, \quad (34d)$$

$$H_{yi} = -iE_0fk_i(I_0 - I_2\cos 2\varphi)/\eta, \quad (34e)$$

and

$$H_{z_t} = -2E_0 f k_t I_1 \sin\varphi/\eta. \tag{34f}$$

The locally polarized waves are now functions of the diffraction integrals and are obtained by substituting the E_i terms from Eqs. (34) into Eqs. (19). Equations (21) and (22) then give the local energy fluences in each wave, and Eqs. (24) and (25) give the absorptivities.

Scattering is also treated in a similar way. Equations (31) and (32) give the surface currents in terms of $\mathbf{e}_i$ and $\mathbf{h}_i$, Eqs. (28) and (29) give the reflection coefficients, and Eq. (30) gives the scattered field.

The analogy between our analysis and that of Ref. 2 is as follows. First, the far-field amplitude for the scattered field, $\mathbf{F}'(\theta_s', \varphi_s'|\theta_i', \varphi_i')$, of Ref. 2 can be expressed from our analysis in terms of the surface currents, Eqs. (31) and (32), as will be shown in a forthcoming paper. Second, the scatterer matrix $T_{rr'}$ of Ref. 2, which contains the eight surface integrals of spherical vector harmonics, is equivalent to the two coordinate-dependent reflection coefficients in Eqs. (28) and (29), but the inclusion of the scatterer shape and physical properties in the R terms is simpler and more explicit than that in the $T_{rr'}$.

7. CONCLUSION

We generalized the solution for a plane wave incident upon a plane-conducting surface[10] to the case of arbitrary surface and wave-front shapes. Analytical expressions are obtained for the local refractive index, the absorptivities, and the reflection coefficients for a finitely conducting arbitrary surface. The explicit inclusion of the surface function and its derivatives into our solution makes it possible to follow the time history of the laser–solid interaction process when thermodynamic changes take place. The solution obtained is useful in computing the scattering, as well as the extinction parameters, for a wide range of surface geometries. Our solution is also simpler than the methods of solution by the Laplace transform, the Debye potentials, and the vector wave equation.[2,16,17] Finally, since the surface integrals of both the e and the h terms are devoid of spherical vector harmonics, the computation time is expected to be greatly reduced compared with that of the method described in Ref. 2.

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